\* In the last blog, we have seen the various concepts like two tailed and one tailed (left and right) tests, critical or significant value, randomized and non-randomized test, number of independent observations in a set i.e. degrees of freedom (d.f.), steps of solving testing of hypothesis problems, best or optimum test and its properties.

Now we will see *optimum test under different situations*. So

*Continuing the concept*…..

* ***Most Powerful Test (MP Test)*** :- Let Ω be the sample space and C be the critical region or rejection region and C’= Ω - C be the acceptance region.

A test *T* of H0 : Ɵ = Ɵ0 vs H1 : Ɵ = Ɵ1(simple vs simple) based on the sample values : (x1, x2, ….. , xn) is said to be a *Most Powerful Test* if and only if,

P[ ϵ C | H0] = ∫C L0 dx = α

and for any other test *Tn* of the same size α,

PT[ ϵ C | H1] ≥ PTn[ ϵ C1 | H1]

where C1 satisfies the condition,

∫C1 L0 dx = α

The critical region α satisfying the above conditions is known as the *most powerful critical region* of size α.

* ***Uniformly Most Powerful Test*** :- Let us now take up the case of testing a simple null hypothesis against a composite alternative hypothesis, e.g., for testing

H0 : Ɵ = Ɵ0

vs

H1 : Ɵ < Ɵ0 or

H1 : Ɵ > Ɵ0 or

H1 : Ɵ ≠ Ɵ0

In such a case, for a predetermined α, the best test for H0 is called the *uniformly most powerful test of level* α.

A test of *T* of H0 : Ɵ = Ɵ0 vs H1 : Ɵ ≠ Ɵ0 is said to be *Uniformly Most Powerful Test* of size α if,

P[ ϵ C | H0] = ∫C L0 dx = α

and PT[ ϵ C | H1] ≥ PT[ ϵ C1 | H1] for all Ɵ ≠ Ɵ0 where C1 is anu other critical region such that,

∫C1 L0 dx = α

The critical region α satisfying the above conditions is called *uniformly most powerful* (UMP) *region*.

**Note** :- For a two-tailed test, *UMP Test* does not exist except U[0,Ɵ].

In short, *UMP Test* exist for H0 : Ɵ = Ɵ0 vs H1 : Ɵ1 (< Ɵ0) as well as H0 : Ɵ = Ɵ0 vs H1 : Ɵ1 (> Ɵ0) but no *UMP Test* exists for testing H0 : Ɵ = Ɵ0 vs H1 : Ɵ ≠ Ɵ0.

* ***Neyman-Pearson Lemma (NP Lemma)*** :- The lemma given by *J. Neyman* and *E.S. Pearson* provides the most powerful test for testing a simple null hypothesis against a simple alternative hypothesis.

Let X1, X2, ….. ,Xn be a random sample from a density f(,Ɵ) where Ɵ ϵ Θ, the parametric space. Θ consists only two elements Ɵ0 and Ɵ1, i.e., Θ = {Ɵ0 , Ɵ1}. Also, the joint probability density function of X1, X2, ….. ,Xn.

L(,Ɵ) = f(x1,Ɵ) f(x2,Ɵ), ….., f(xn,Ɵ)

L(,Ɵ) is the likelihood function of sample observations for a parameter value of Ɵ where = x1, x2, ….., xn.

If there exists a critical region C of size α and a non-negative number K such that

L(,Ɵ1)/L(,Ɵ0) > K for every x ϵ C

and L(,Ɵ1)/L(,Ɵ0) ≤ K for every x ∉ C

Then C is said to be the best critical region (BCR) of size α for testing H0 : Ɵ = Ɵ0 vs H1 : Ɵ = Ɵ1.

Any test *T* corresponding to BCR C is the most powerful test of size α of the hypothesis H0 against H1.

Neyman-Pearson Lemma helps to determine the size of type I and type II errors for the given range of x. Also, it helps to find out the power and power function of a test in case of testing a simple null hypothesis against a simple alternative hypothesis.

* ***Minimax Test*** :- A test *T* of H0 : Ɵ ϵ Ɵ0 vs H1 : Ɵ ϵ Ɵ1 is said to be a *Minimax Test* if for any other Tn, the following inequality holds.

Max[R(T,Ɵ0), R(T,Ɵ1)] ≤ Max[R(Tn,Ɵ0), R(Tn,Ɵ1)]

Where R(T,Ɵ) is the risk in using the test *T* when Ɵ is the true value of the parameter, So is R(Tn,Ɵ).

* ***Unbiased Test and Unbiased Critical Region*** :- A test *T* of the null hypothesis H0 : Ɵ ϵ Θ0 vs H1 : Ɵ ϵ Θ1 is said to be an unbiased test if the probability of rejecting H0 when it is false at least as much as the probability of rejecting H0 when it is true.

Let us consider the testing of H0 : Ɵ = Ɵ0 vs H1 : Ɵ = Ɵ1. The critical region C and consequently the test based on it is said to be unbiased if the power of the test exceeds the size of the critical region, i.e., if

Power of the test ≥ Size of the C.R.

1 - β ≥ α

P[ ϵ C | H1] ≥ P[ ϵ C | H0]

In other words, the critical region C is said to be unbiased if

PƟ(C) ≥ PƟ0(C), for all Ɵ (≠Ɵ) ϵ Θ